Irreducible Representations

representation $\varphi : G \longrightarrow GL(V)$

is irreducible if $V \neq \{0\}$

and the only invariant subspaces

are $\{0\}$ and V.

Example: Every 1-dimensional rep.

is irreducible

Example: $\rho : S_3 \longrightarrow GL_3(\mathbb{C})$ std rep

not irreducible

$W = \mathbb{C}v, \quad v = e_1 + e_2 + e_3$ $\longrightarrow$ invariant subspaces

$U = \mathbb{C}x + \mathbb{C}y, \quad \underbrace{x = e_1 - e_2, \quad y = e_2 - e_3}$

$\Rightarrow$ $(\varphi|_W)$ and $(\varphi|_U)$

are irreducible S_3-reps.

$(\rho|_U) \sim \varphi : S_3 \to GL_2(\mathbb{C})$

$$\varphi_{(12)} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \varphi_{(123)} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$$

<u>Claim</u>: φ irreducible. $\dim \varphi = 2$

Consider $H = \mathbb{C}z$, $z \in \mathbb{C}^2 \setminus \{0\}$

H invariant $\iff \varphi_{(12)}(z), \varphi_{(123)}(z) \in \mathbb{C}z$

i.e., z is an eigenvector of both matrices

$\varphi_{(12)} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$ has eigenvectors $\underbrace{\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$

$\varphi_{(123)} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$ not eigenvectors